Trigonometry, Mckeague, 6th edition
Homework Proof

5.4 #49

The EASY way requires using an identity that you have **not** memorized.

Prove: $\tan\left(\frac{B}{2}\right) = \csc B - \cot B.$

Start with:

$$\tan\left(\frac{B}{2}\right)$$

$$= \frac{1-\cos B}{\sin B}$$

$$= \frac{1}{\sin B} - \frac{\cos B}{\sin B}$$

$$= \csc B - \cot B.$$

Therefore, $\tan\left(\frac{B}{2}\right) = \csc B - \cot B.$

See below on page 2

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The HARDER way can be done with identities that you have memorized.

Prove: $\tan\left(\frac{B}{2}\right) = \csc B - \cot B.$

Start with:

$$\tan\left(\frac{B}{2}\right) = \frac{\sin\left(\frac{B}{2}\right)}{\cos\left(\frac{B}{2}\right)}$$

I need some identities to move forward, but I don't have much.
Also, I'd like to get rid of those half angles
which means I need double angle identites.
I'll force one by multiple by a "fancy one".

$$= \frac{\sin\left(\frac{B}{2}\right)}{\cos\left(\frac{B}{2}\right)} \cdot \left(\frac{\sin\left(\frac{B}{2}\right)}{\sin\left(\frac{B}{2}\right)}\right) = \frac{\sin^2\left(\frac{B}{2}\right)}{\sin\left(\frac{B}{2}\right)\cos\left(\frac{B}{2}\right)}$$

Notice the bottom is almost like the double angle of sine.
It just needs a factor of 2 which I can multiply.

$$= \frac{\sin^2\left(\frac{B}{2}\right)}{\sin\left(\frac{B}{2}\right)\cos\left(\frac{B}{2}\right)} \cdot \left(\frac{2}{2}\right) = \frac{2\sin^2\left(\frac{B}{2}\right)}{2\sin\left(\frac{B}{2}\right)\cos\left(\frac{B}{2}\right)} = \frac{2\sin^2\left(\frac{B}{2}\right)}{\sin\left(2\left(\frac{B}{2}\right)\right)} = \frac{2\sin^2\left(\frac{B}{2}\right)}{\sin B}$$

$$= \frac{2\sin^2\left(\frac{B}{2}\right)}{\sin B}$$ Now the hard part. How to get rid of the half angle on top.

I am going to need to force in a double angle identity again,
But instead of multiplying by a "fancy one", I am going to add a "fancy zero".

$$= \frac{1-1+2\sin^2\left(\frac{B}{2}\right)}{\sin B} = \frac{1-\left(1-2\sin^2\left(\frac{B}{2}\right)\right)}{\sin B} = \frac{1-\cos\left(2\cdot\left(\frac{B}{2}\right)\right)}{\sin B} = \frac{1-\cos B}{\sin B}$$

$$= \frac{1-\cos B}{\sin B} = \frac{1}{\sin B} - \frac{\cos B}{\sin B} = \csc B - \cot B.$$

Therefore, $\tan\left(\frac{B}{2}\right) = \csc B - \cot B.$

Well, like I said. This is the harder way, but study it and learn some of those techniques I used to force identities to come up (e.g. the fancy one, the fancy zero).