Trigonometry, Mckeague, 6th edition
Homework Proof

5.2 #57

Prove: $\sec(A+B)=\dfrac{\cos(A-B)}{\cos^2 A-\sin^2 B}$.

Start with:

$\sec(A+B)$

$=\dfrac{1}{\cos(A+B)}$

$=\dfrac{1}{\cos(A+B)}\left(\dfrac{\cos(A-B)}{\cos(A-B)}\right)$ Note : the use of this "fancy one" converts the top into what I want.

$=\dfrac{\cos(A-B)}{\cos(A+B)\cos(A-B)}$

$=\dfrac{\cos(A-B)}{(\cos A\cos B-\sin A\sin B)(\cos A\cos B+\sin A\sin B)}$ Now FOIL

$=\dfrac{\cos(A-B)}{\cos^2 A\cos^2 B-\sin A\sin B\cos A\cos B+\sin A\sin B\cos A\cos B-\sin^2 A\sin^2 B}$

$=\dfrac{\cos(A-B)}{\cos^2 A\cos^2 B-\sin^2 A\sin^2 B}$

The next step is a bit tricky, but I don't want $\cos^2 B$ or $\sin^2 A$ in the bottom,

so I need something to change them out. I'll use the Pythagrean Identities.

$=\dfrac{\cos(A-B)}{\cos^2 A(1-\sin^2 B)-(1-\cos^2 A)\sin^2 B}$

$=\dfrac{\cos(A-B)}{\cos^2 A-\cos^2 A\sin^2 B-\sin^2 B+\cos^2 A\sin^2 B}$

$=\dfrac{\cos(A-B)}{\cos^2 A-\sin^2 B}$.

Therefore, $\sec(A+B)=\dfrac{\cos(A-B)}{\cos^2 A-\sin^2 B}$