Math 90 Lecture Notes
Chapter 7

Section 7.1: Systems of Equations and Graphing

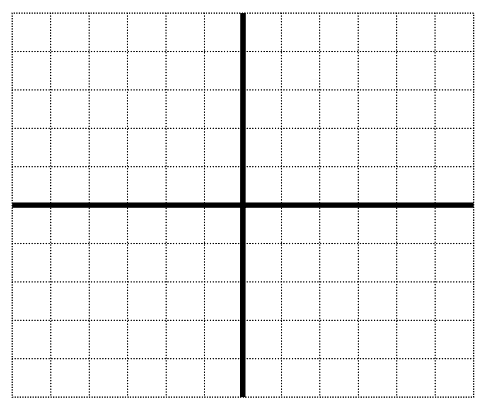
In this chapter we will learn three ways to solve what is known as a system of linear equations. In many situations, it is helpful and necessary to use more than one equation to model a real world situation. We then solve this "system" of equations to find our solution. Read the following example to see how this can work.

In this section we will also study the three types of systems. See pages 432 and 433 in the text book.

a) Consistent-Independent---> Lines that cross
b) Consistent-dependent---> Lines laying on top of each other
c) Inconsistent-independent---> Parallel lines

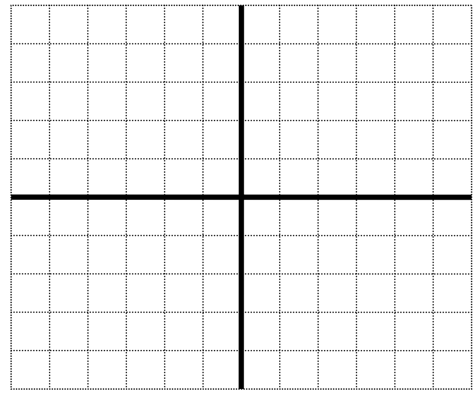
Dependent: One equation is a multiple of the other.
Consistent: A pair of equations that result in only one solution.

A. In this section, we will use the method of Graphing to solve systems of linear equations.
   1. Solve the following system by graphing:
      $x + y = 4$
      $x - y = -2$

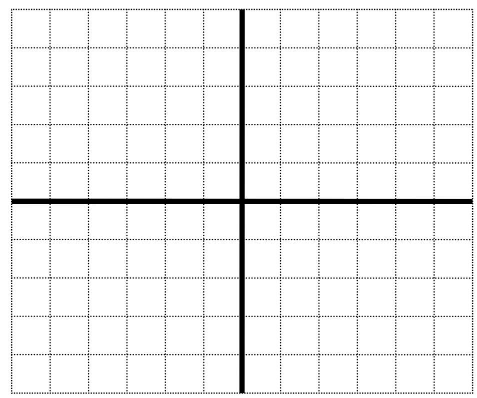


      a. Note: Since each equation we are graphing is linear, its graph will be a line.
      b. We are looking for the point where the graphs intersect. This is a point that they both share and is our solution.

2. Solve the following system of equations:
   $2x - 4y = 8$
   $2x - y = -1$



3. Solve the following system of equations:
   $y = \frac{2}{3}x + 2$ and $y = \frac{2}{3}x - 3$



4. The example given above shows that in some cases, there are no solutions to a system of equations. This occurs in cases when the lines represented by those equations that are parallel.

5. It is also possible for a system of equations to have an infinite number of solutions. Look at the following system to see how this can happen. Solve the following: $2x + y = 4$ and $4x + 2y = 8$

B: **DO THESE PROBLEMS**

Solve the sets of simultaneous equations, in the following manner, by graphing.

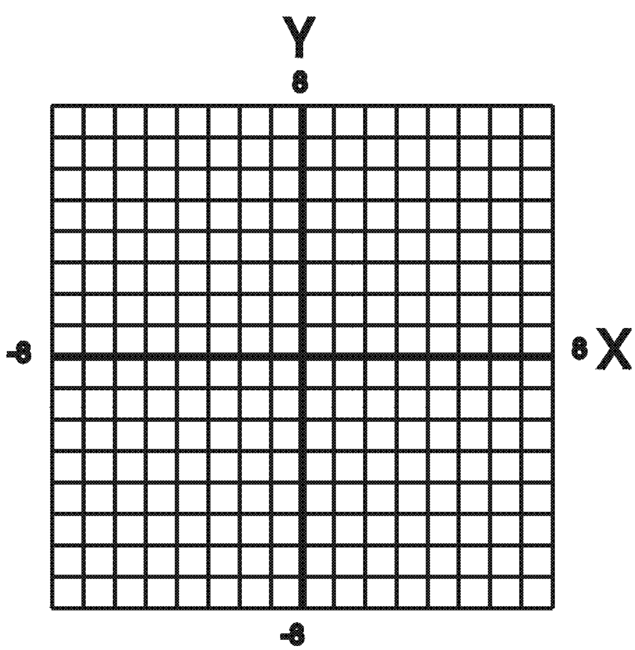
1. For each value of x given, compute y and enter it in the table.
2. Plot the coordinates given in the tables.
3. The resulting points should all lie on a straight line.
4. Do this for each equation.
5. Plot the two equations on the left on the left grid; plot the two equations on the right on the right grid.

$y = x + 4$

X	**Y**
0	
-2	
+2	
-4	
+4	

$y = -2x - 4$

X	**Y**
0	
-2	
2	
-4	

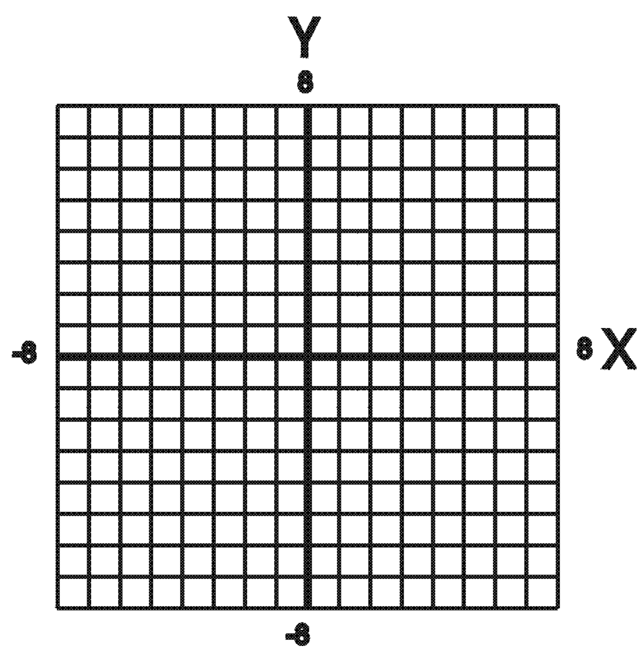


$2x + y = 8$

X	**Y**
0	
2	
3	
4	

$x - y = 7$

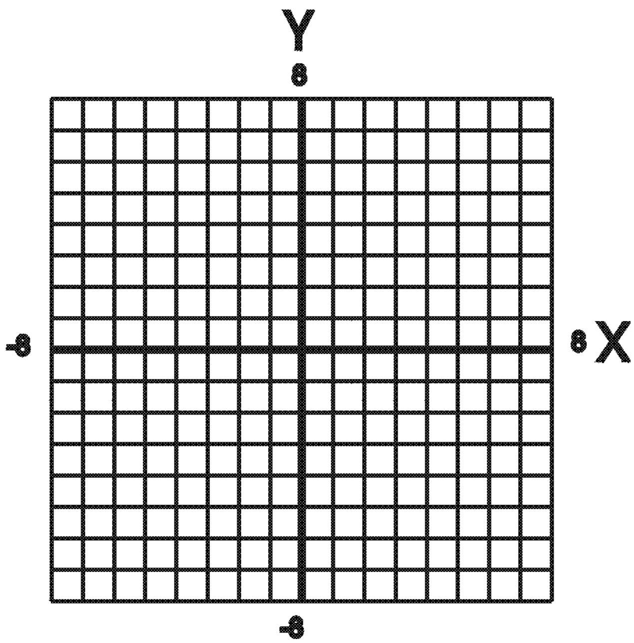
X	**Y**
0	
2	
3	
4	
-1	

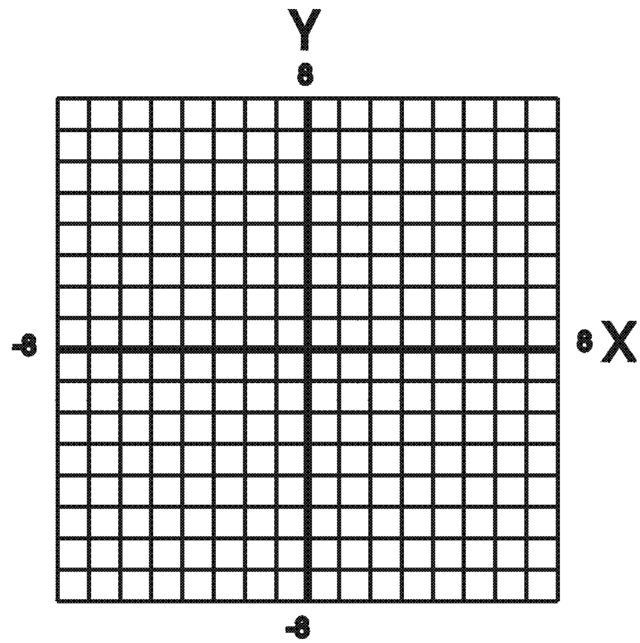


In each of the following problems, graph both equations (by any method) on a single grid. If there is a common solution to the two equations, the two straight lines will intersect and the point of intersection is the solution. If the lines intersect, write the point of intersection as an ordered pair. If the lines do not intersect, there is no solution. If the lines are coincident (lie one on top of the other) there are an infinite number of solutions.

4. $\begin{array}{l} 4x - 20 = 5y \\ 8x - 10y = 12 \end{array}$

5. $\begin{array}{l} y = \frac{1}{3}x + 2 \\ 3y = x - 21 \end{array}$





Section 7.2: Systems of Equations and Substitution

The method of substitution for solving systems of linear equations works for any linear system, but works best if one of the equations in the system can easily be solved for one of the variables. What we look for in particular is a variable in either equation that has a coefficient of 1. This makes the substitution method very easy to use. Work through the following examples to see how easy this method is to use. Remember that we are trying to find a point where the two lines intersect, and a point has both an x and a y coordinate.

A. Some sample problems: Solve using the method of Substitution.

1. $2x + y = 1$
   $x - 5y = 17$

2. $2x - 4y = -4$
   $x + 2y = 8$

3. $7x + 6y = -9$
   $y = -2x + 1$

B. Remember that not all systems of linear equations have one unique solution. What are the other cases?

1. If the lines are parallel, how many solutions will you find? ______

2. If the lines coincide, how many solutions will you find? _______ .

C. Some rules for solving systems of linear equations

5. If possible, remove by factoring, any constant multiplier that may be in the equations. This action will not change the equality but will make the equation simpler.
6. Re-arrange the equations such that both are in the same form. For instance place all terms on the left with zero on the right.
7. If the two equations are identical, then these equations lie one on top of the other and there are an infinite number of solutions.
8. If the variable terms are identical but the constants different then these are parallel lines and there is no solution.
9. If options 3 and 4 are both false then there is one solution.

D. Work through the following examples to see what happens when you encounter one of the situations described above:

1. $2x + 6y = -18$ How many solutions? _____________
   $x + 3y = -9$

2. $10x + 2y = -6$ How many solutions? _____________
   $y = -5x$

E. Some examples of application problems:

1. Find two numbers such that the sum is 76 and the difference is 12.

2. Two angles are supplementary. One angle is 8 degrees less than three times the other. Find the measure of each angle.

3. The state of Wyoming is a rectangle with a perimeter of 1280 miles. The width is 90 miles less than the length. Find the length and the width.

Section 7.3: Systems of Equations and Elimination

In this section we will learn another method for solving systems of equations called the Elimination Method. In this method, you want to ultimately add the two equations together and eliminate one of the variables. Work through the following examples to see how this is done.

A. The Rational behind this method: Note the following:

$$\begin{array}{l} \text{If } A = B \text{ and} \\ \underline{\quad C = D} \text{ then it is true that: } A+C = B+D \end{array}$$

Because of this fact, we can always add two equations together to get another true equation. We will use this fact when we use the elimination method for solving systems of equations.

B. Some sample problems: Solve the following using the Elimination Method. But before solving these equations let us examine them to determine what kind of solution we can expect.

1. $x - y = 1$
   $-x - y = -7$

2. $x - y = 4$
   $2x + y = 8$

3. $-x + 10y = 1$
   $-5x + 15y = -9$

4. $2x + 9y = 2$
   $5x + 3y = -8$

C. Once again remember that not all systems of equations have a unique solution. Follow the next two examples to see what happens when we get no solution, or an infinite number of solutions.

1. $8x - 2y = 2$
   $4x - y = 2$ How many solutions does this system have?

2. $$\frac{-1}{3}x - \frac{1}{2}y = -\frac{2}{3}$$
   $$-\frac{2}{3}x - y = -\frac{4}{3}$$
   How many solutions does this system have?

Section 7.4: More Applications using Systems

In this section we will practice solving application problems using systems of equations. The most important part of this process is setting up the system that models the situation. You can use either the substitution method or the elimination method to solve the system that you set up.

A. Sample application problems:

1. In winning the 2000 conference finals, the Los Angeles Lakers scored 69 of their points on a combination of 31 two and three pointers. How many of each type of shot did they make?

2. Filmworks charges $1.75 for a role of 24-exposure film and $2.25 for a 36-exposure roll of film. Stu bought 19 rolls of film for a total of $39.25. How many rolls of each type did he buy?

3. From November 2 through January 3, the Bronx Zoo charges $6.00 for adults and $3.00 for children and seniors. One December day, a total of $1554 was collected from 394 admissions. How many adult and how many children/senior admissions were there?

4. Sunflower seed is worth \$1.00 per pound and rolled oats are worth \$1.35 per pound. How much of each would you use to make 50 pounds of a mixture worth \$1.14 per pound.

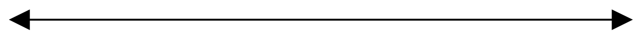
5. Clear Shine window cleaner is 12% alcohol and Sunstream window cleaner is 30% alcohol. How many ounces of each should be used to make 90 ounces of a cleaner that is 20% alcohol?

Section 7.5: Linear Inequalities in Two Variables

In this section we will learn how to find solutions of linear inequalities in two variables.

A. An example of an inequality in one variable. Solve and graph the solution set for the following inequality:

$$3(x+5) \le 22$$

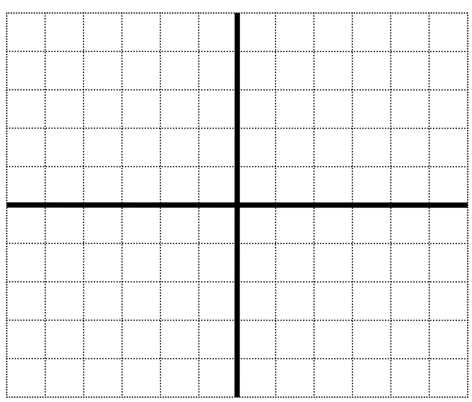


Note: There are an infinite number of solutions to this inequality. Because of this we need to describe our solutions and/or graph them as it would be impossible to list them all.

B. Find and graph as many ordered pair solutions to the following inequality.

$$y \le x + 3$$

X	Y



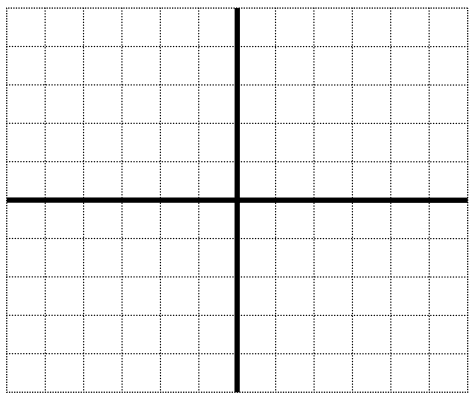
C. What do you notice about the boundary for the solutions to this inequality in two variables?

1. ____

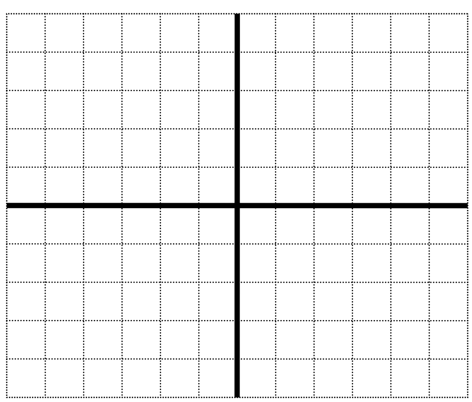
2. ____

D. Graph the solution set for the following inequalities in two variables:

1. $y \le 3x + 2$



2. $5x + 4y \ge 20$



Remember that when you are graphing the solution set to an inequality in two variables that you must first graph the "Boundary Line". The boundary line will be solid when: ______________________ , and the boundary line will be dashed or dotted when: _______________________ .