

Math 90 Lecture Notes

Chapter 3

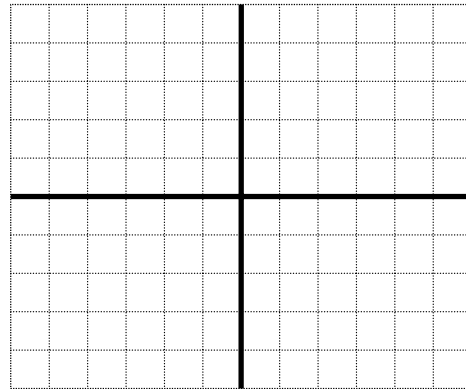
Section 3.1: Reading Graphs, Plotting Points, and Estimating Values

In this section we will concentrate on graphing ordered pairs in the Rectangular Coordinate System, and take a quick look at a process known as Interpolation and Extrapolation.

A. Graphing Ordered Pairs in the Rectangular Coordinate Plane:

1. Label the following on the Coordinate Plane:

- The x-axis
- The y-axis
- The Origin
- The 4 Quadrants



2. Points on the plane can be represented as “Ordered Pairs”.

- The first number in an ordered pair is called the _____ coordinate .
- The second number in an ordered pair is called the _____ coordinate.
- An ordered pair is basically giving you directions from the Origin to another location in the plane. The x-coordinate tells you how far to go from the _____ axis, and the y-coordinate tells you how far to go from the _____ axis .

3. Graph the following points on the axis above: $(2, 5)$, $(-2, 1)$, $(-2, -5)$, $(3, -3)$

4: List the Coordinates for the points graphed below:

1: _____

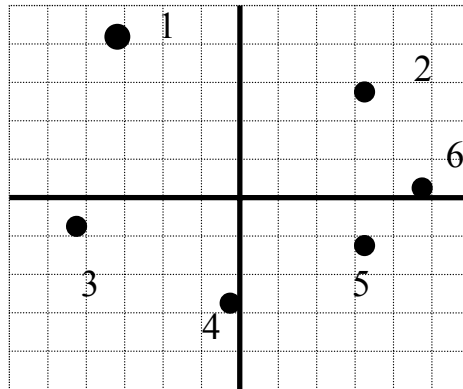
2: _____

3: _____

4: _____

5: _____

6: _____



B. DO THESE EXCERSIZES WITH ORDERED PAIRS

1: In which quadrant is each ordered pair located:

(7,-2) _____

(-1,-4) _____

(-4,6) _____

(7.5,2.9) _____

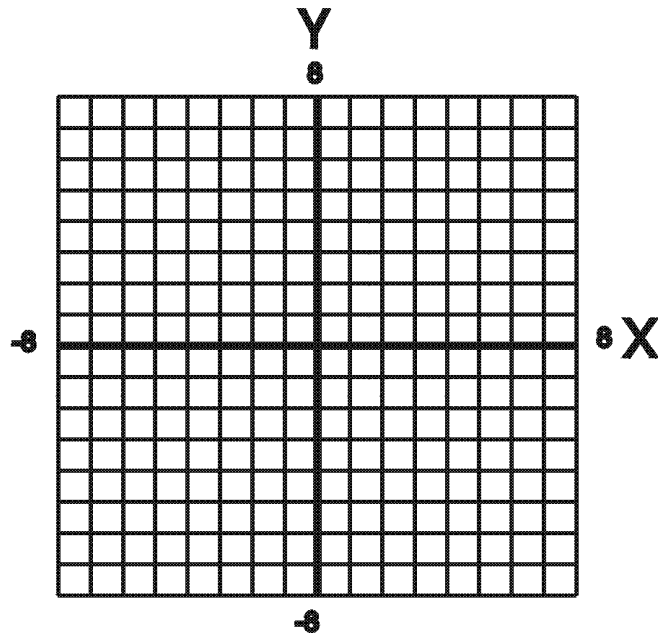
2: Plot, and label, the following ordered pairs

A: (-7,-4)

B: (0,4)

C: (6,0)

D: (5, -3)

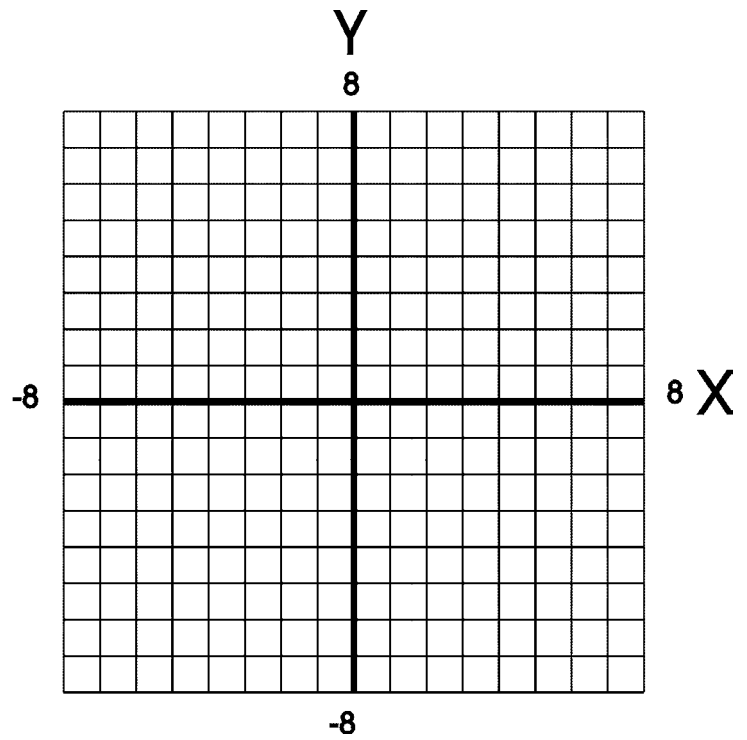


3: The following ordered pairs are three corners of a rectangle. Plot these pairs and draw the rectangle.

$(5,-2), (-3,-2), (-3,3)$

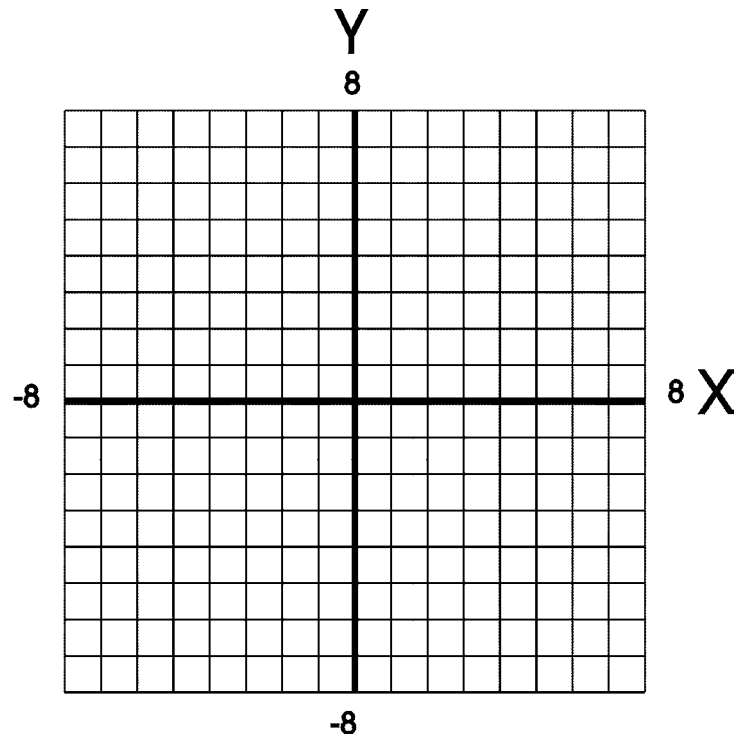
a) What is the missing ordered pair:

b) What is the perimeter of the rectangle.



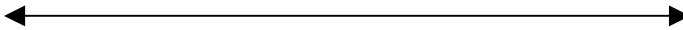
4: Find the area of a triangle defined by the following ordered pairs.

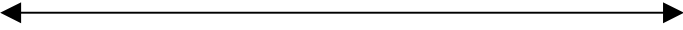
$(0,8), (0,-4), (5,-4)$



Section 3.2: Graphing Linear Equations

A. Review: Finding and graphing solutions to equations in one variable. Solve and graph the solutions to the following:

1. $5x + 25 = 50$ 

2. $-3x + 21 = 33$ 

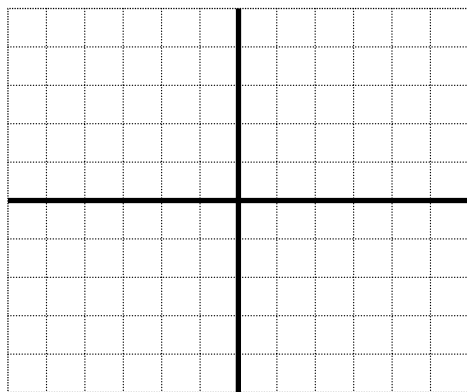
B. Finding solutions for equations with two variables:

- Note that in both cases above, there was one variable and only one solution for each equation. This is often the case in equations with only one variable.
- Find three possible solutions for the following equation: You must pick a value for one of the variables then solve for the other variable.

$$2x + y = 10$$

y	x	

3. Graph each point found above on the coordinate axis below:



What do you notice about the points you have graphed?

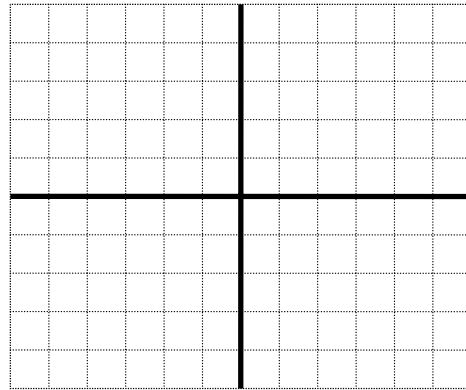
C. How to determine if a point is a solution to a given equation: Plug in the coordinates and then determine if you end up with a true statement. Work through the examples below to practice this skill.

1. Is the point $(0, 3)$ a solution to the following equation? $y = 2x + 3$

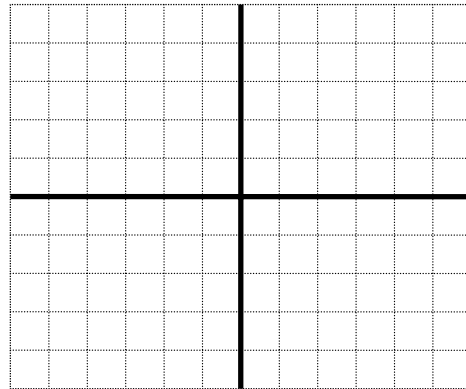
2. Is the point $(0, 5)$ a solution to the following equation? $5x - 3y = 15$

D. Graph the following equations by using the “Point-Plotting Method”.

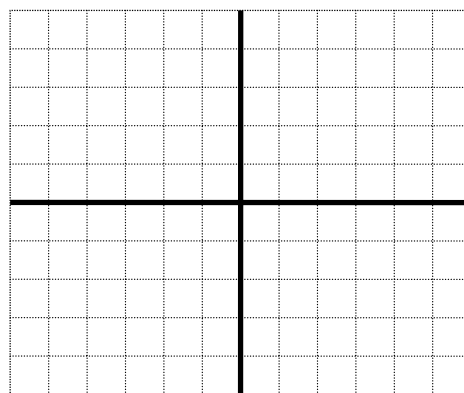
1. $Y = 2x + 2$



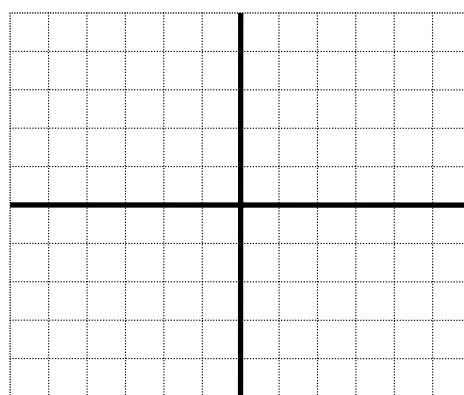
2. $y = \frac{1}{3}x$



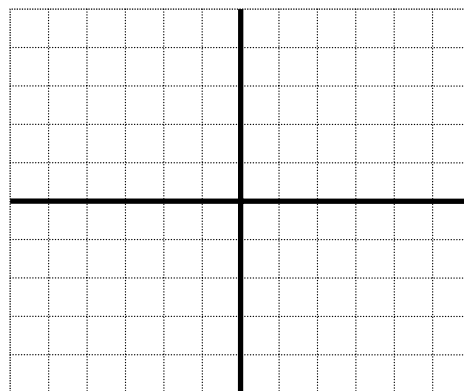
3. $y = \frac{5}{2}x + 3$



4. $x + 2y = -6$



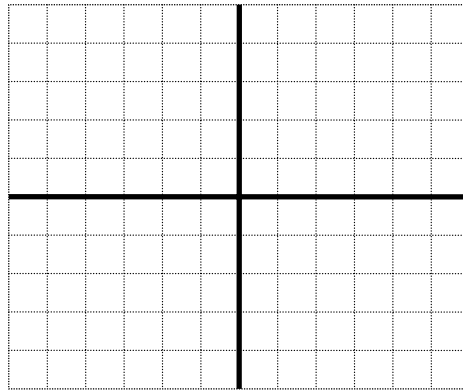
5. $8x - 4y = 12$



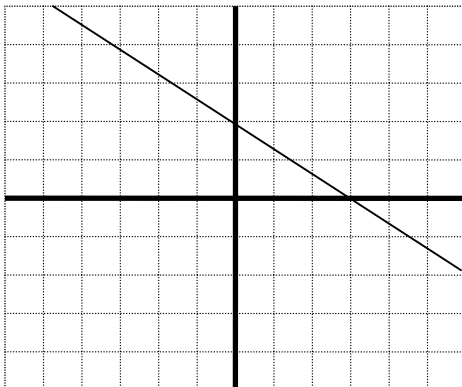
Section 3.3: Graphing and Intercepts

In this section we will learn how to graph linear equations using another method, actually an application of the Point-Plotting Method, called the Intercept Method. We will also learn how to graph horizontal and vertical lines.

- A. Intercepts: The “x-intercept” is the point where a graph crosses the _____ axis. The “y-intercept” is the point where the graph crosses the _____ axis.
- B. Sketch a graph below and point out the x and y-intercepts. Note that the x-coordinate of the y-intercept is always _____. Also note that the y-coordinate of the x-intercept is always _____.

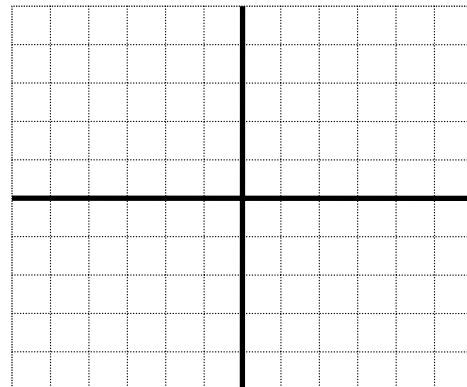


Find the x and y intercepts of the line in the graph.



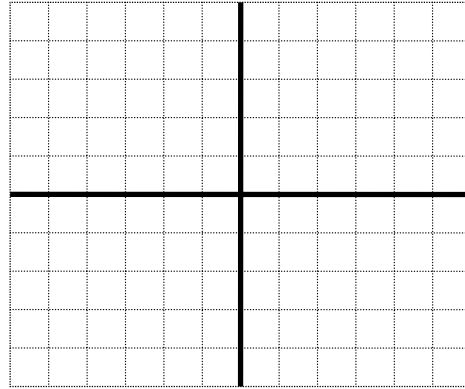
Plot this equation

$$3x - 2y = 6$$

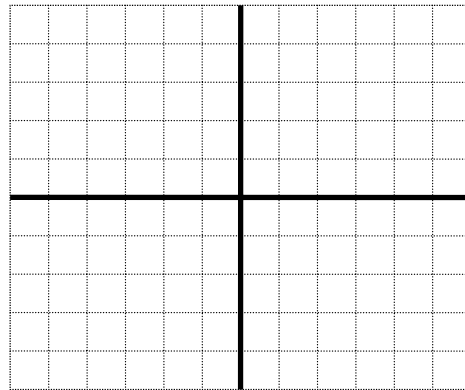


PLOT THESE EQUATIONS

3. $y = 2x - 6$

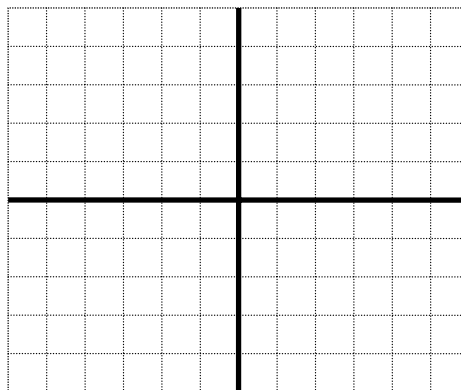


4. $3x - y = 2$

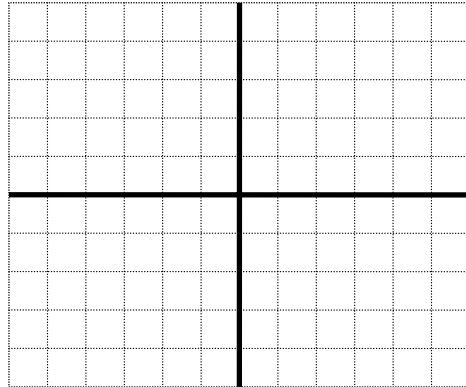


Graphing Vertical and Horizontal Lines: These lines are a little different than the ones you are used to graphing from previous problems.

5: Graph $y = 3$

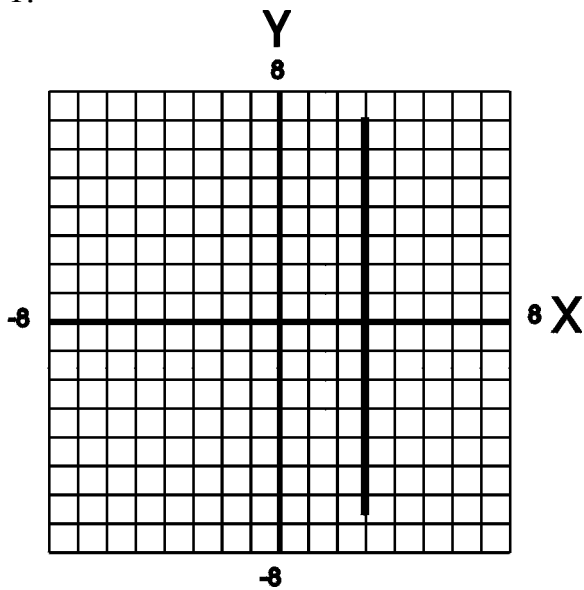


6: Graph $x = -3$

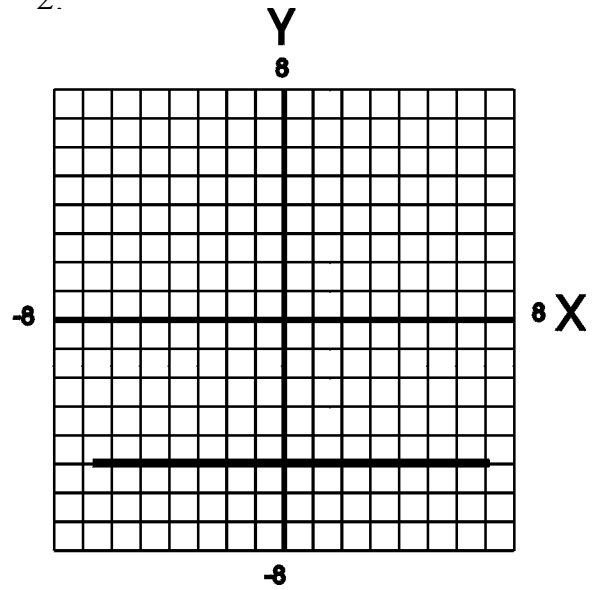


C: Write an equation for each of the following two graphs.

1.



2.



Chapter 3, Section 4: Rates of Change

In this section we will study how to measure rates of change. This is a very important application of slope as we continue our study of graphs.

A. Rates of Change: A **“Rate”** is a ratio that indicates how two quantities change with respect to each other! Discuss some familiar “rates” of change that you deal with in everyday life:

1. Speed in Miles per hour.
- 2.
- 3.

B. DO THESE PROBLEMS

1: On February 10, Oscar rented a Chevy Blazer with a full tank of gas and 13,091 miles on the odometer. On February 12, he returned the vehicle with 13,322 miles on the odometer. The rental agency charged \$92 for the rental and needed 14 gallons of gas to fill the tank.

a) Find the Blazer’s rate of gas consumption, in miles per gallon.

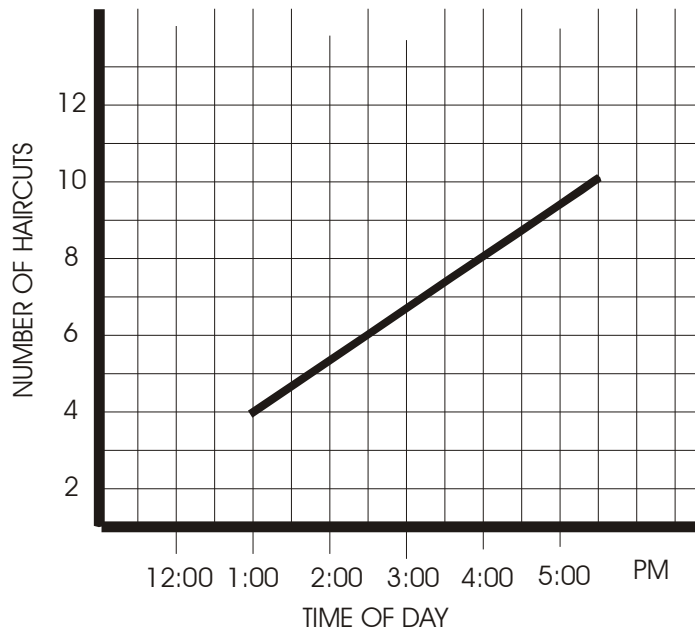
b) Find the average cost of the rental, in dollars per day.

c) Find the rate of travel, in miles per day.

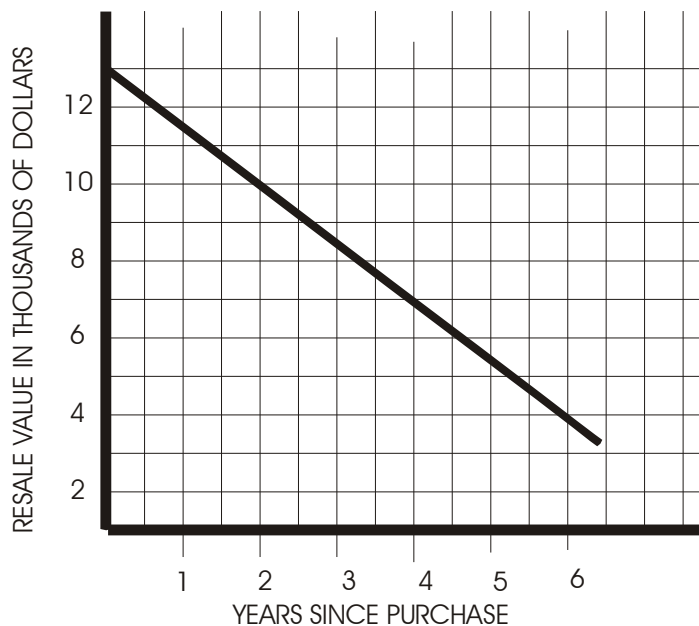
d) Find the rental rate, in cents per mile.

2: The tuition at a college was \$1327 in 1999. In 2001 the tuition was \$1359. What is the rate at which tuition is increasing?

3: Eve's Custom Cuts has a graph displaying data from a recent days work. At what rate does Eve work? Note that the hours scale on the horizontal axis cannot be used directly but must be converted to a time **difference**.



4: The graph below depicts the value of a car in thousands of dollars versus year since purchase. What is the rate of depreciation? Don't forget the algebraic sign.



Section 3.5: Slope

In this section we will continue our study of slope. Slope and “rate of change” are closely related. In fact, mathematically they are the same. The difference is that “rate of change” always has units (e.g. Miles/gallon) associated with it whereas slope may or may not have associated units.

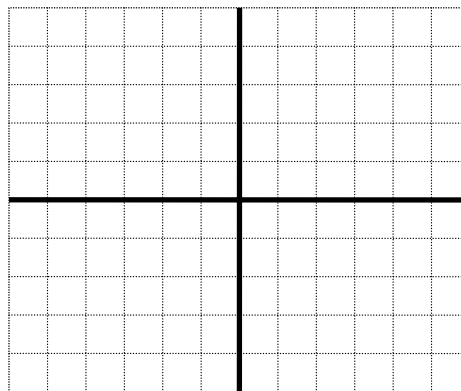
A. **VERY IMPORTANT:** Definition of slope:

1. Slope as a rate of change of the y-coordinate compared to the x-coordinate.
2. Slope = $m = \frac{\text{Rise}}{\text{Run}}$
3. Slope = $m = \frac{\Delta y}{\Delta x}$
4. Slope = $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$

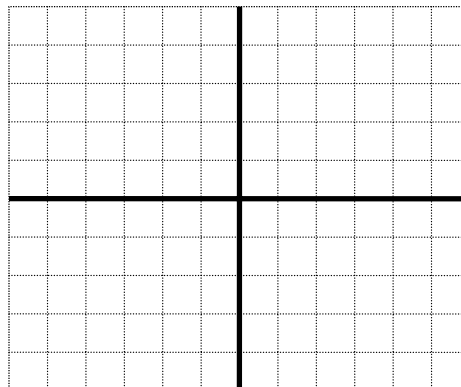
B. DO THESE PROBLEMS:

Find the slope of the line containing each given pair of points. If the slope is undefined, state this. Use the grids to check your answers.

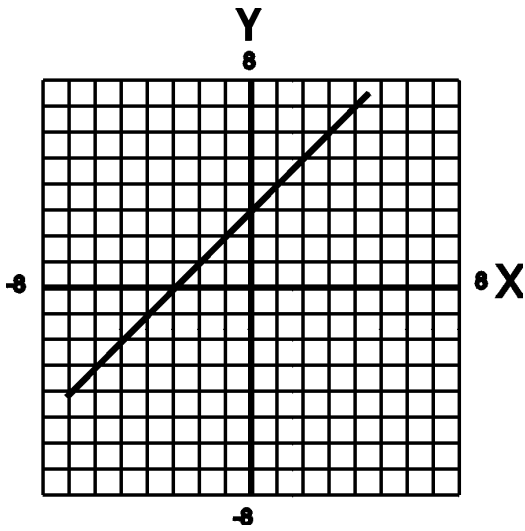
1. (2 , 1) and (−5 , 3)



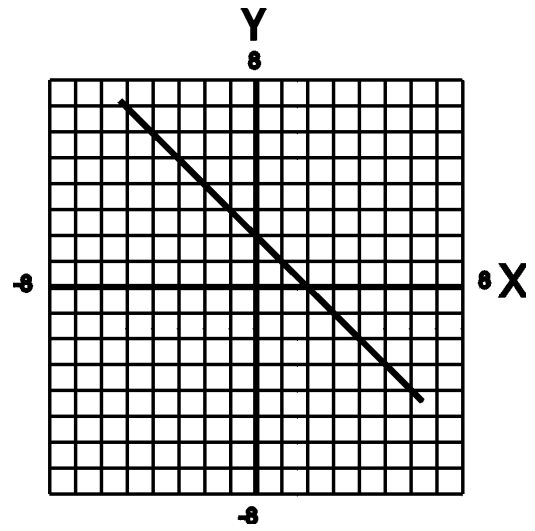
2. (−10 , 3) and (−10 , 4)



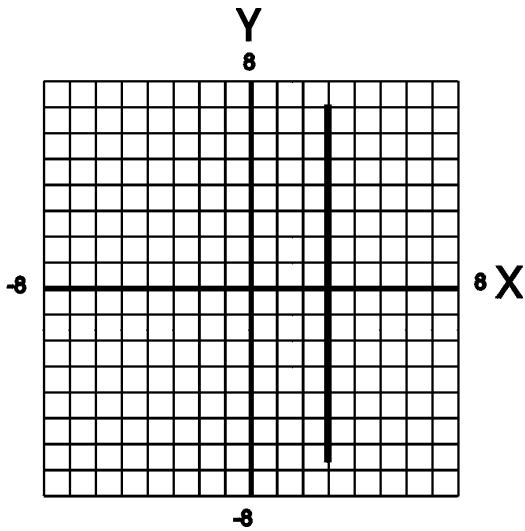
5: Write the slope of the line below each graph.



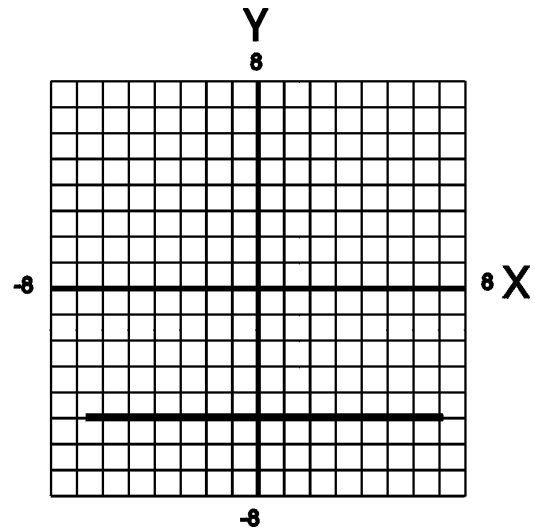
SLOPE: _____



SLOPE: _____



SLOPE: _____

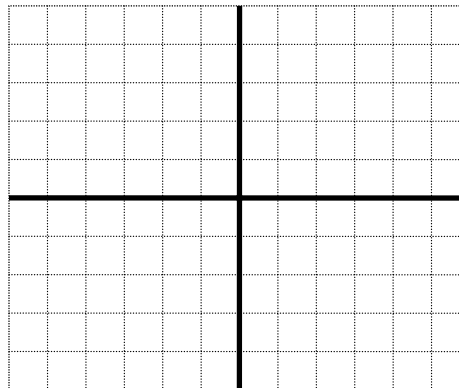


SLOPE: _____

Section 3.6: Slope-Intercept Form of the Equation

In this section we will learn how to use our knowledge of slope to simplify the graphing process.

- A. Graph the following equation: $3y - 2x = -6$



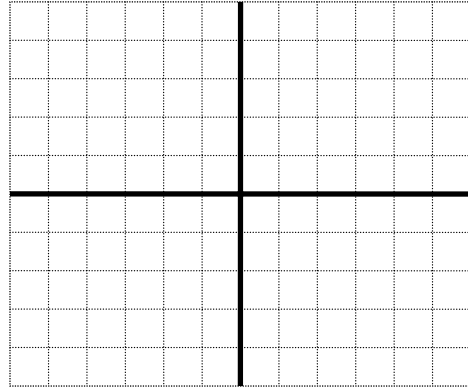
1. What is the slope of this line? _____
 2. What is the y-intercept for this line? _____
 3. Solve the above equation for y:
-
4. Note the y-intercept and slope in this equation!
- B. The ***“Slope-Intercept”*** form of the equation:
1. $Y = mx + b$
 2. “m” represents the _____ of the line.
 3. “b” represent the _____ of the line.
- C. If the SLOPE is the same in a pair of equations then the lines plotted by those equations are PARALLEL. **Circle the constants in each equation telling us these two lines are parallel.**

$$y = -2x + 3; \quad y = -2x - 4$$

D. DO THESE PROBLEMS

1. Draw a line that has the given slope and y-intercept:

Slope = $m = \frac{2}{5}$, and y-intercept (0, -3)



2. Find the slope and y-intercept for the following equation:

$$y = \frac{-3}{8}x + 6$$

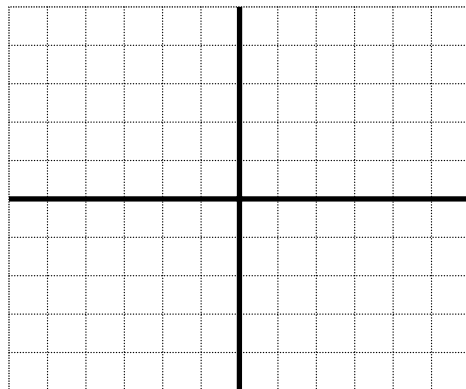
3. Find the slope and y-intercept for the following equation:

$$3x - 2y = 18$$

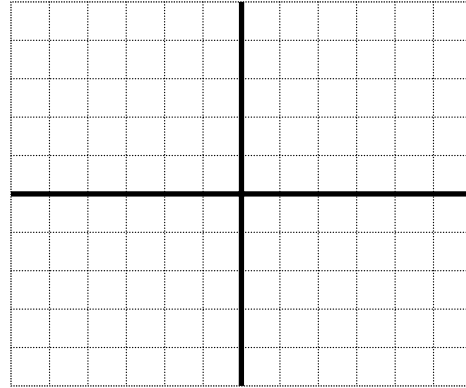
4. Graph the line with the following equation:

$$y = \frac{-3}{5}x - 1$$

$$y = \frac{-3}{5}x + 2$$



5. Graph the line with the given equation:
 $4x + 5y = 15$ (First put equation in Slope-Intercept Form)



In the following problems, find the equation with the given slope and y-intercept:

6. Slope = $m = -4$, and y-intercept $(0, -2)$

7. Slope = $m = \frac{3}{4}$, and y-intercept $(0, 23)$

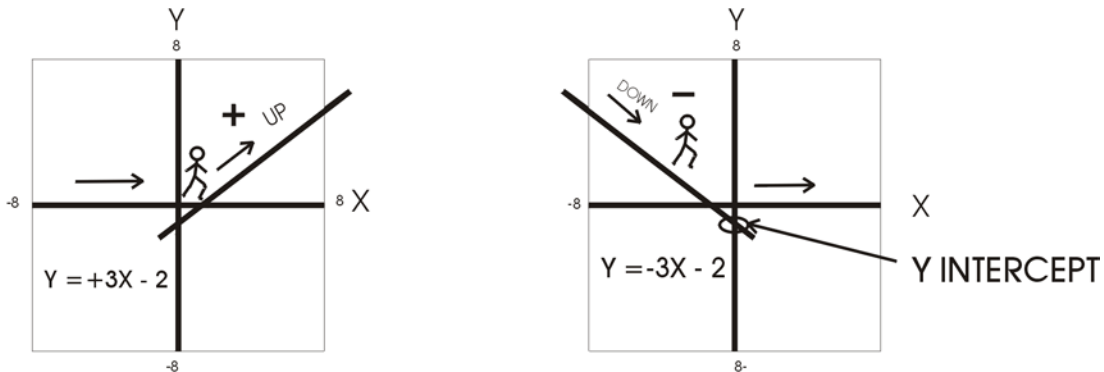
GETTING IT RIGHT

GRAPHS

1. REWRITE THE EQUATION IN SLOPE INTERCEPT FORM

$$Y = 3X - 2$$

↑ ↙
SLOPE Y INTERCEPT



SLOPE: VISUALIZE A PERSON WALKING FROM LEFT TO RIGHT, JUST AS YOU READ, AND MEETING A HILL GOING UPHILL OR DOWNHILL.

UP-POSITIVE DOWN-NEGATIVE ← **MEMORIZE**

3. USING THE ORIGINAL EQUATION, PREPARE A TABLE OF X AND Y VALUES. THE TABLE SHOULD HAVE THREE POINTS.

FIRST POINT: LET $X=0$; CALCULATE Y
SECOND POINT: LET $Y=0$; CALCULATE X
THIRD POINT: PICK A NUMBER FOR X; CALCULATE Y

X	Y
0	-2
$\frac{2}{3}$	0
2	4

$Y = +3X - 2$

4. PLOT THE THREE POINTS. THEY SHOULD LIE ON A STRAIGHT LINE.

5. CHECK THE SLOPE AND Y INTERCEPT OF THE LINE AGAINST THE RESULTS YOU OBTAINED IN STEP 1.

Section 3.7: Point-Slope Form of a Linear Equation

In this section we will learn how to write equations of lines based on some information about that line. This is a very important skill, so learn it well.

A. Writing Equations of Lines: What do you need?

1. You need a _____ that the line goes through.
2. You need to know the _____ of the line.
3. If the above information is not known, you need to find it!

B.

1. When the unknown point is located between two known points, the process is known as "**Interpolation**".
2. When the unknown point is located Outside two known points, the process is known as "**Extrapolation**".

C. Work through the following problems to practice writing equations:

1. Write the equation of the line that goes through the following point, with the following slope: $(6, 2)$, slope = $m = 3$

2. Write the equation of the line that goes through the following point, with the following slope: $(4, 7)$, slope = $m = \frac{3}{2}$

3. Write the equation of the line that goes through the following two points: (3 , 7) and (4 , 8)

4. Write the equation of the line that goes through the following two points: (-2 , 3) and (2 , 5)

5: Write a point slope equation for the line with given slope and given point then plot the equation.

$$m = \frac{5}{4}; (-2, 6)$$

