Chapter 2

Section 2.1: Solving Equations

In this section we will use the "***Addition Principle***" and the ***"Multiplication Principle"*** to solve equations.

A. How to check if a solution really works for a given equation:

1. If y = 26 does x = 7? 3x + 5 = y?

2. If $y=\frac{8}{3}$, does $x=\frac{2}{3}$? $x+\frac{1}{3}=y$

3. **ALWAYS be sure to check your answers in the back of the book, or by plugging your solution into the original equation.**

B. The Addition and Multiplication Principles:
1. The ***Addition Principle*** states that you can _________________ the same number from/to both sides of an equation.
2. Record a couple of examples to demonstrate this fact:

3. The **Multiplication Principle** states that you can _____________ the same number on both sides of an equation.
4. Record a couple of examples to demonstrate this fact:

C. In solving equations, basically we are changing the original equation into successively simpler or equivalent equations that eventually lead us to an answer!

D. When solving equations, we are striving to isolate the variable on the left side of the equation with constants on the right side.

E. DO THESE PROBLEMS:

1. $y + 9 = 43$

2. $-6 = x - 11$

3. $t + \frac{3}{8} = \frac{5}{8}$

4. $y - \frac{3}{4} = \frac{5}{6}$

5. $\frac{y}{-8} = 11$

6. $\frac{-x}{6} = 9$

7. $-\frac{2}{5}x = -\frac{4}{15}$

8. $-0.2344x = 2028.732$

Section 2.2: Using the Principles Together

In this section we will use both the Addition and Multiplication Principles together in solving equations. We will also learn how to simplify an equation before using these two principles.

A. Some equations that involve both principles:

1. $3x + 6 = 30$
2. $-6z - 18 = -132$

B. Some equations that involve simplification before applying the principles: Note the summary of equation solving on page 94 of your text.

1. $6x + 19x = 100$
2. $8(2t + 1) = 4(7t + 7)$

C. Solving equations that involve Fractions: The beauty of solving equations is that you can "get rid of" any fractions in the equation. This then makes the solution process much easier! Work through the following examples to see how this process works.

1. $\frac{1}{2} + 4m = 3m - \frac{5}{2}$
2. $1 - \frac{2}{3}y = \frac{9}{5} - \frac{1}{5}y + \frac{3}{5}$

D. We can eliminate decimals from equations by following a similar process as we used to eliminate fractions. However, with the use of our calculators, we can usually solve decimal equations without the need to eliminate the decimals. (Note the next example)

1. $0.91 - 0.2z = 1.23 - 0.6z$

E. DO THESE PROBLEMS

1. $4x+3=-21$

2. $4+\frac{7}{2}x=-10$

3. $7+3x-6=3x+5-x$

4. $7(5x-2)=6(6x-1)$

5. $0.7(3x+6)=1.1-(x+2)$

6. A good way to solve this equation is to start by clearing fractions. How do you clear fractions?

$$\frac{1}{6}\left(\frac{3}{4}x-2\right)=-\frac{1}{5}$$

Section 2.3: Formulas

Many applications of mathematics are found in the use of Formulas. In this section we will explore how to use formulas to solve many types of problems.

A. Some sample problems that involve using formulas:

1. The Power rating "P", in Watts, of an electrical appliance is determined by the following formula: $P = I \cdot V$, where "I" is the current, in amperes, and "V" is the voltage, measured in volts. If a kitchen requires 30 amps of current and the voltage in the house is 115 volts, what is the wattage of the kitchen?

2. The surface area "A" of a cube with sides of length "s" is given by the following formula: $A = 6s^2$. Find the surface area of a cube with sides of 3 inches.

3. When all "n" teams in a league play every other team twice, a total of "N" games are played, where $N = n^2 - n$. If a soccer league has 7 teams and all teams play each other twice, how many games are played?

B. Many times it is helpful to solve a formula for a specific variable. The next few problems will give examples of how to do this.

1. Solve the following formula for "w": P = 2L + 2W

2. Solve the following for "c": $A = \frac{a+b+c}{3}$

3. Solve the following for "x": S = Ax + Bx

C. DO THESE PROBLEMS:

1. The area of a parallelogram is given by the formula: $A = b \bullet h$.
Assume you know both the area, A, and the height, h. What is formula for the base , b, in terms of area and height?

2. The velocity of sound is approximately 1000 feet/second in air. You are watching the Space Shuttle take off at Cape Canaveral. You see the ignition and eleven seconds later you hear the roar. How far away is the launch pad if the equation for the velocity of sound C, is distance , D, divided by time T. First solve the following equation for D.

$$C = \frac{D}{T}$$

3. The formula $A = P + Prt$ is used to compute the amount of money [A] in an account which is earning interest at rate [r] over a period of time [t]. Solve this formula for principle, [P].

Section 2.4: Applications with Percent

Working with Percents is something we all do as a natural part of our lives. In this section we will review how to use percents. See page 106 in your book.

A. Some important facts about percents:

1. PERCENT means parts per hundred. The phrase CENT refers to 100.

2. This little formula may be helpful $\frac{P}{100} = \frac{Sample}{Population} = \frac{IS}{OF}$

Here P is Percent; Sample is a part of the Population. Note, the "Population" is merely a reference and the "Sample" may be larger or smaller than the population. Examine the problem. Sometimes $\frac{Sample}{Population}$ is the best; sometimes $\frac{IS}{OF}$ is more useful depending on how the problem is worded.

EXAMPLE: In a certain group of 120 people, 15 are children. What Percent of this group are children?

The Sample is 15 and the population is 120 so:

ANS: $\frac{P}{100} = \frac{15}{120}$ So $P = \frac{15}{120} \bullet 100 = 12.5\%$

3. The fraction $\frac{P}{100}$ is the decimal equivalent of the Percent, P. So

$$Decimal\ Equiv = \frac{P}{100}$$

Example: Change 25% to a decimal: $\frac{25}{100} = 0.25$

4. And to convert a decimal to a Percent we rewrite the above formula:

$$P = Decimal \bullet 100$$

Example: Convert .45 to a percent: $P = 0.45 \bullet 100 = 45\%$

B. **PROBLEMS:** Working sample problems that involve percents. Usually you can make a direct translation from the problem in English to Mathematics. Work through the next few problems to see how this is done.

1. What percent of 150 is 39?

2. 54 is 24% of what number?

3. What number is 1% of one million?

4. Convert to decimal notation: 125%

5. Convert to % notation: $\frac{4}{5}$

6. 7 is 175% of what number?

7. What is 2% of 40?

8. To obtain his degree, Frank must complete 125 hours (credits) of instruction. If he has already completed 60% of this requirement, how many more credits will he need?

Section 2.5: Problem Solving

One of the most important reasons for studying Algebra, is to use it in solving problems. This section will allow you to practice this very important skill. A mathematical word problem should not be read as though it is a story or as though it conveys some kind of useful information. A math word problem is a word image describing a relation amongst several elements. When ever possible use the statements in the problem to draw a picture then go to the problem to get information about the relevant elements of the picture.

A. The Five Steps to Problem Solving: Page 115
   1. Familiarize yourself with the problem: This may mean that you have to read the problem several times until you figure out what you need to do, and what information you have to do it! **ALWAYS: If possible, draw a picture!!** Underline the key words.
   2. Translate to Mathematical language. (This usually means that you define a variable to represent what you are trying to find and then write an equation that fits the situation)
   3. Carry out some mathematical manipulation. (In other words, solve the equation you created)
   4. Check your possible answer in the original problem. (Does you answer make sense in the given situation?)
   5. State the answer clearly. (Be sure to answer the given question. Don't just say that $x = 5$)

B. Look at the suggestions for becoming familiar with the problem on page 115 of your text. These are some very good suggestions as to how you might approach a problem solving situation.

C. **PROBLEMS:** Some sample problems for practice: (Be sure to read this section of your text very carefully and follow all of their examples as well)

   1. Twice the sum of 4 and some numbers is 34. What is the number?

   2. Doug paid $72 for a shockproof portable CD player during a 20% off sale. What was the original price?

3. The Iditarod sled-dog race extends for 1049 miles from Anchorage to Nome. If a musher is twice as far from Anchorage as from Nome, how many miles has the musher traveled?

4. The sum of two consecutive odd integers is 108. What are the integers?

5. In the world's oldest divorcing couple, the woman was 6 years younger than the man. Together, their ages totaled 188 years. How old were the man and the woman?

6. The second angle of a triangle is four times as large as the first. The third angle is 5 degrees more than the sum of the other two angles. Find the measure of the second angle. (Note: The sum of the angles of any triangle is always 180 degrees)

7. The top of the John Hancock Building in Chicago is a rectangle whose length is 60 feet more than the width. The perimeter is 520 feet. Find the width and the length of the rectangle. Find the area of the rectangle.

Section 2.6: Solving Inequalities

In this section we will learn how to solve an inequality. Basically you solve them just as you would an equation with one major exception that will be described below. See page 127 in your book.

A. Addition Property of Inequalities: You can add or subtract the same number from any inequality. Record some examples below to show why this is true.

1.

3.

2.

4.

B. Multiplication Property of Inequalities:

1. You can multiply or divide both sides of an inequality ***by any positive number***. Record some examples below.

2. IF you multiply or divide both sides of an inequality by a negative number, you must change the direction of the inequality. Record some examples below.

C. **PROBLEMS:** Graphing inequalities: Because inequalities usually have an infinite number of solutions, we often graph the solution sets. Practice this on the examples below. Also see page 128.

1: Graph $x > -2$

2: Graph $x \le 5$

Solve and graph the solution sets for the following inequalities. Also write the solutions sets in set builder notation.

3: $y + 6 > 9$

4: $2x + 4 \le x + 1$

5: $7 + 8x \ge 71$

6: $\frac{2}{3} - \frac{x}{5} < \frac{4}{15}$

7: $3(t - 2) \ge 9(t + 2)$

8: $\frac{4}{5}(3x + 4) \le 20$