Cheat Sheet – Exam 3

Derivatives

1. $\dfrac{d}{dx}(\tan x) = \sec^2 x$

2. $\dfrac{d}{dx}(\cot x) = -\csc^2 x$

3. $\dfrac{d}{dx}(\sec x) = \sec x \tan x$

4. $\dfrac{d}{dx}(\csc x) = -\csc x \cot x$

5. $\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}$

6. $\dfrac{d}{dx}(\cos^{-1} x) = -\dfrac{1}{\sqrt{1-x^2}}$

7. $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$

8. $\dfrac{d}{dx}(\cot^{-1} x) = -\dfrac{1}{1+x^2}$

9. $\dfrac{d}{dx}(\sec^{-1} x) = \dfrac{1}{x\sqrt{x^2-1}}$

10. $\dfrac{d}{dx}(\csc^{-1} x) = -\dfrac{1}{x\sqrt{x^2-1}}$

11. $\dfrac{d}{dx}(\sinh x) = \cosh x$

12. $\dfrac{d}{dx}(\cosh x) = \sinh x$

Integrals

13. $\displaystyle \int \dfrac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\dfrac{x}{a}\right) + C$

14. $\displaystyle \int \dfrac{1}{a^2+x^2}\,dx = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right) + C$

15. $\displaystyle \int \dfrac{1}{x\sqrt{x^2-a^2}}\,dx = \dfrac{1}{a}\sec^{-1}\left(\dfrac{x}{a}\right) + C$

16. $\displaystyle \int \ln x\,dx = x\ln x - x + C$

17. $\displaystyle \int \tan x\,dx = \ln|\sec x| + C$

18. $\displaystyle \int \sec x\,dx = \ln|\sec x + \tan x| + C$

19. $\displaystyle \int \cot x\,dx = -\ln|\csc x| + C$

20. $\displaystyle \int \csc x\,dx = \ln|\csc x - \cot x| + C$

21. $\displaystyle \int \sec^3 x\,dx = \dfrac{\sec x \tan x}{2} + \dfrac{\ln|\sec x + \tan x|}{2} + C$

Trig Identities

22. $\sin^2 x + \cos^2 x = 1$

23. $1 + \tan^2 x = \sec^2 x$

24. $1 + \cot^2 x = \csc^2 x$

25. $\sin 2x = 2\sin x \cos x$

26. $\cos 2x = \cos^2 x - \sin^2 x$

27. $\sin A \cos B = \tfrac{1}{2}[\sin(A+B) + \sin(A-B)]$

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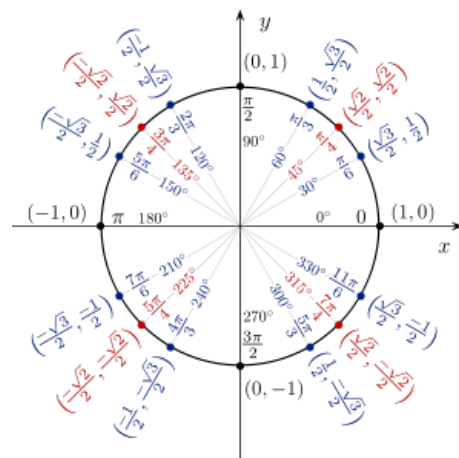
29. $\sin A \sin B = \tfrac{1}{2}[\cos(A-B) + \cos(A+B)]$

Right Angle Trigonometry

30. $\sin\theta = \dfrac{opp}{hyp} \qquad \csc\theta = \dfrac{hyp}{opp}$

31. $\cos\theta = \dfrac{adj}{hyp} \qquad \sec\theta = \dfrac{hyp}{adj}$

32. $\tan\theta = \dfrac{opp}{adj} \qquad \cot\theta = \dfrac{adj}{opp}$

Half-Angle Formulas

33. $\sin^2 x = \tfrac{1}{2}(1 - \cos 2x)$

34. $\cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin x$	0	$\tfrac{1}{2}$	$\tfrac{\sqrt{2}}{2}$	$\tfrac{\sqrt{3}}{2}$	1
$\cos x$	1	$\tfrac{\sqrt{3}}{2}$	$\tfrac{\sqrt{2}}{2}$	$\tfrac{1}{2}$	0
$\tan x$	0	$\tfrac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undef.

7.1 Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

7.3 Trig Substitution

$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

Parametric Equations

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \qquad\qquad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy/dt}{dx/dt}\right)}{dx/dt}$$

Area between curve and x-axis:

$$\int_\alpha^\beta y\,dx = \int_\alpha^\beta g(t) f'(t)\,dt$$

Arc Length: $L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Polar Equations

$$x = r\cos\theta \qquad\qquad r^2 = x^2 + y^2$$

$$y = r\sin\theta \qquad\qquad \tan\theta = \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

Area inside the curve: $\int_\alpha^\beta \frac{1}{2} r^2\,d\theta$

Area between curves: $\int_\alpha^\beta \frac{1}{2}\left(r_1^2 - r_2^2\right)d\theta$

Arc Length: $L = \int_\alpha^\beta \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\,d\theta$

Function

Arc Length: $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$

L'Hospital's Rule:

If $\dfrac{f(x)}{g(x)} \to \dfrac{0}{0}$ or $\dfrac{f(x)}{g(x)} \to \dfrac{\infty}{\infty}$, then

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)}$$

11.3 Error bounds for Integral Test

If f is continuous, positive and decreasing, then the error is bounded by:

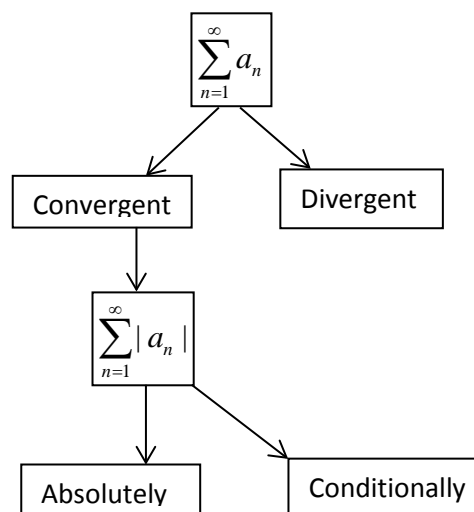$$\int_{n+1}^\infty f(x)\,dx \le R_n \le \int_n^\infty f(x)\,dx \text{ and}$$

$$S_n + \int_{n+1}^\infty f(x)\,dx \le S \le S_n + \int_n^\infty f(x)\,dx$$

11.5 Error bounds for Alternating Series Test

If $\displaystyle\sum_{n=1}^\infty (-1)^n b_n$ is an alternating series, then

$$|R_n| \le b_{n+1} \text{ and } S_n \le S \le S_{n+1} \text{ or } S_{n+1} \le S \le S_n$$

Absolute Convergence

$$\sum_{n=1}^\infty a_n$$

Convergent   Divergent

$$\sum_{n=1}^\infty |a_n|$$

Absolutely   Conditionally

Test	Series	Convergence or Diverges	Comments
Divergence test	$\sum a_n$	Diverges if $\lim\limits_{n\to\infty} a_n \neq 0$	Inconclusive if $\lim\limits_{n\to\infty} a_n = 0$
Geometric series	$\sum\limits_{n=0}^{\infty} ar^n$ or $\sum\limits_{n=1}^{\infty} ar^{n-1}$	Converges to $\dfrac{a}{1-r}$ only if $|r| < 1$ Diverges if $|r| \geq 1$	Useful for comparison tests if the nth term a_n of a series is similar to ar^n.
p-series	$\sum\limits_{n=1}^{\infty} \dfrac{1}{n^p}$	Converges if $p > 1$ Diverges if $p \leq 1$	Useful for comparison tests if the nth term a_n of a series is similar to $\dfrac{1}{n^p}$.
Integral	$\sum\limits_{n=c}^{\infty} a_n \ (c \geq 0)$ $a_n = f(n) \ for \ all \ n$	Converges if $\int_c^{\infty} f(x)\, dx \ converges$ Diverges if $\int_c^{\infty} f(x)\, dx \ diverges$	The function f obtained from $a_n = f(n)$ must be continuous, positive, decreasing and readily integrable for $x \geq c$.
Comparison	$\sum a_n$ and $\sum b_n$	If $\sum b_n$ converges and $0 \leq a_n \leq b_n$, then $\sum a_n$ converges If $\sum b_n$ diverges and $0 \leq b_n \leq a_n$, then $\sum a_n$ diverges	The comparison series $\sum b_n$ is often a geometric series or a *p*-series.
Limit comparison	$\sum a_n$ and $\sum b_n$ With $a_n, b_n > 0$ for all n And $\lim\limits_{n\to\infty} \dfrac{a_n}{b_n} = L > 0$	$\sum b_n$ converges $\to \sum a_n$ converges $\sum b_n$ diverges $\to \sum a_n$ diverges	The comparison series $\sum b_n$ is often a geometric series or a *p*-series. To find b_n consider only the terms of a_n that have the greatest effect on the magnitude.
Ratio	$\sum a_n$ with $\lim\limits_{n\to\infty} \dfrac{|a_n+1|}{|a_n|} = L$	Converges (absolutely) if L< 1 Diverges if L > 1 or if L is infinite	Inconclusive if L = 1. Useful if a_n involves factorials or n^{th} powers.
Root	$\sum a_n$ with $\lim\limits_{n\to\infty} \sqrt[n]{|a_n|} = L$	Converges (absolutely) if L< 1 Diverges if L > 1 or if L is infinite	Test is Inconclusive if L = 1. Useful if a_n involves n^{th} powers.
Absolute Value $\sum |a_n|$	$\sum a_n$	$\sum |a_n|$ converges $\to \sum a_n$ converges	Useful for series containing both positive and negative terms.
Alternating series	$\sum\limits_{n=1}^{\infty} (-1)^{n-1} b_n \ (b_n > 0)$	Converges if $0 < b_{n+1} < b_n$ for all n and $\lim\limits_{n\to\infty} b_n = 0$	Applicable only to series with alternating terms.